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LOCAL STABILITY ANALYSIS OF PASSIVE DYNAMIC BIPEDALROBOT

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ABSTRACT

The bipedal walking is the main form of locomotion of human kind. The human body is flexible and so it is easy for humankind to steadily walk on the different terrain, howeverbuilding arobot to have a human-like gait is not easy due to the complex dynamics of the walking.In this paper, we focus the passive dynamic bipedal robot (PDBR) which walks only by thepresentof gravity on an inclined ramp, that is, the robotis walking on a ramp in absence of external forces. We describe ageneral method for deriving the equations of motion and impactequations for the analysis of walkingmodels. Application of Poincare fixed point method to analyse the stability of a symmetric gait after the small disturbance in the fixed point. It used to understand, how much disturbance is allowed for the stable cyclic motion.

KEYWORDS: Bipedal Robot, Passive Walking, Linearization, Switched Conditions, Compass Gait, Orbital Stability, Poincare Map, Phase Space Diagram

INTRODUCTION

The study of walking of bipedal robots is inspired by the walking of human body. However the making robot to get human-like gait is not easy because of the complex dynamics of the walking. We focus on the study of walking motion of biped robots without knee. Sever approaches are available in the literature. Mochon and MacMahon[1] showed that bipedal walking is a passive mechanical process. McGeer[2] expanded the idea and explained that a completely unactuated and therefore uncontrolled robot can execute a stable walk on the inclined ramp. Gracia[3] investigated that the effect of slope angle of ramp on the walking motion of the passive robot.He found thatif the slope of the ramp decreases, walking speed also decreases and increasing the slope angle carries about a period doubling bifurcation leading to chaotic gaits and there are just unstable gaits in high speed region. Collins, Wisse and Ruina[4] established 3D passive-dynamic robot with kneesand arms. Passive-dynamic walking is useful for learning efficient level-ground walking robots, but it has some limitationsdescribed by [5]. Goswami [6] have carried out the extensivesimulation analyses of the stability of theunactuated biped walker. However, the bipedrobots are expected not only to walk steadily, but also to walk fast. Many other researchers have studied about this topic [7-10].

In this paper, we studythe dynamics of the passive dynamic walking of simple, 2D, unactuated, two-link model, vaguely resemblinghuman legs without knee, bipedal robot which steps down on the ramp with an angle α. We examine the walking of passive dynamic stepwise by considering jump conditions immediately after the impact of swing leg with the ramp surface and switching the velocities between swing leg and stance leg. We have used Poincare map to find periodic solutionfor thebiped robot (orbit or limit cycle). At this point, we have investigated the stability of PDBR usingthe phase space diagrams and analytic properties of fixed point of the Poincare map.

Mathematical Modeling

This model is the simplest model of passive dynamic robots, consisting two rigid mass-less legs hinged atthe hip, a point-mass at the hip, and infinitesimal point-masses at the feet. The feet have plastic (no-slip. no-bounce) collisions with the ramp, except during forward swinging, when geometric interference (foot scuffing) is ignored.

The mathematical modeling of biped robots is necessarily hybrid, consisting of ordinary differential equations to describe the swing phase of the walking motion, and a discrete map to model the impact when the leg touches the ground.Biped robots exhibit periodic behavior. Distinct events, such as contact with the ground, can act to trap the evolving system state within a constrained region of the state space. Therefore Limit cycles (periodic behavior) are often created in this way.

Our working model of a passive dynamic robot (PDR) is shown in Figure 1 schematically.

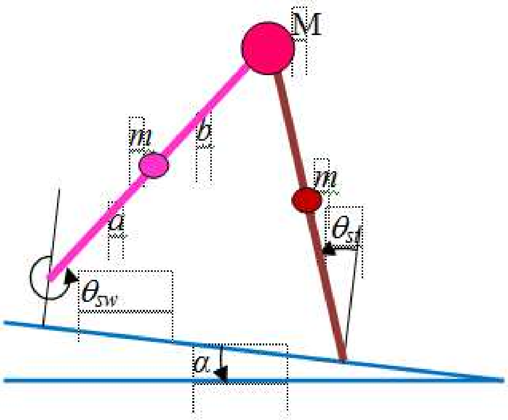

Figure 1: A Passive Dynamic Biped Robot

Table 1: Lists Physical Parameters of this Model

Symbols	Parameters
θ_{st}	Angle of stance leg with vertical upward axis
θ_{sw}	Angle of swing leg with vertical upward axis
a	The length of rod from the CM of Stance (and Swing)leg to the foot
b	The length of rod from the CM of Stance (and Swing)leg to the Hip
l	Length of rod== $a+b$
m	Mass of leg
M	Mass of Hip
α	Slope of ramp
φ	Inter legs Angle

Using Euler-Lagrange equations, the dynamic equation of the robot can be derived as:

$$M(\theta)\ddot{\theta}+N(\theta,\dot{\theta})\dot{\theta}+G(\theta,\alpha)=0 \tag{1}$$

where $\theta=[\theta_{st} \quad \theta_{sw}]^T$, $M(\theta)$ is the inertia matrix, the matrix $N(\theta,\dot{\theta})$ contains terms of centrifugal and coriolis forces, $G(\theta,\alpha)$ is the gravity term as given below:

$$M(\theta) = \begin{bmatrix} ma^2 + Ml^2 + ml^2 & -mlb\cos(\theta_{st} - \theta_{sw}) \\ -mlb\cos(\theta_{st} - \theta_{sw}) & mb^2 \end{bmatrix};$$

$$N(\theta, \dot{\theta}) = \begin{bmatrix} 0 & -mlb\sin(\theta_{st} - \theta_{sw})\dot{\theta}_{sw} \\ mlb\sin(\theta_{st} - \theta_{sw})\dot{\theta}_{st} & 0 \end{bmatrix};$$

$$G(\theta, \alpha) = \begin{bmatrix} (-ma - Ml - ml)g\sin(\theta_{st} - \alpha) \\ mgb\sin(\theta_{sw} - \alpha) \end{bmatrix}$$

Linearized State Space Model

The above non-linear model can be linearized at the equilibrium point $0_e = [\alpha \quad \alpha \quad 0 \quad 0]^T$, the equation becomes

$$M_{0_e}\ddot{\theta} + G_{0_e}\theta = 0$$

So the linerized state space model can be written as

$$\dot{y} = Ay \tag{2}$$

where $y = x - 0_e$, $x = [\theta_{st} \quad \theta_{sw} \quad \dot{\theta}_{st} \quad \dot{\theta}_{sw}]^T$ and the elements of matrix A are:

$$A = \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ -M_{0_e}^{-1}G_{0_e} & 0_{2\times2} \end{bmatrix} \text{ where } M_{0_e}^{-1}G_{0_e} = \begin{bmatrix} \dfrac{(-ma - Ml - ml)g}{(ma^2 + Ml^2)} & \dfrac{mlg}{(ma^2 + Ml^2)} \\ \dfrac{l(-ma - Ml - ml)g}{b(ma^2 + Ml^2)} & \dfrac{(ma^2 + Ml^2 + ml^2)g}{b(ma^2 + Ml^2)} \end{bmatrix}.$$

Heel Strike and Impact Equations

The heel strike occurs when the swing leg touches the ramp surface. We assumed that the heel strike to be inelastic and without slipping, and the stance-leg lifts from theramp without interaction. This impact occurs when the geometric condition, the y-coordinate of a foot of the swing leg will become azero is met, that is,

$$\theta_{st} + \theta_{sw} = 2\alpha.$$

The Angular momentum is conserved at the impact for thewhole robot about the new swing-leg contact foot and for thenew stance-leg about the hip in figure 2.

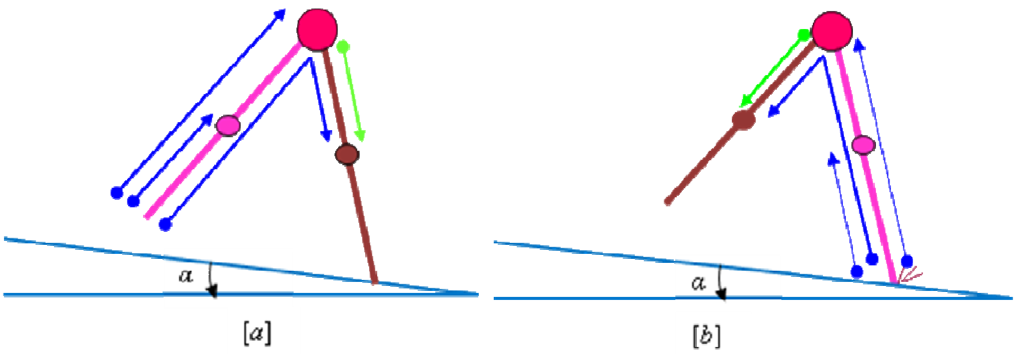

Figure 2 [*a*]: Directions of Angular Momentums about the Swing Leg's Foot and the Hip before the Heel Strike[*b*] Directions of angular momentums about the new stance leg's foot and after the heel strike.

$$\dot{\theta}^{+} = K(\varphi)\dot{\theta}^{-}$$

The conservation law of the angular momentum leads to thefollowing compact equation between the pre- and post-impact angular velocities after the heel strike:

where $\theta_{sw} - \theta_{st} = \varphi$

$$K(\varphi) = \left[V^{+}(\varphi)\right]^{-1} V^{-}(\varphi)$$

where

$$V^{-}(\varphi) = \begin{bmatrix} -mab + \left(2mla + Ml^{2}\right)\cos\varphi & -mab \\ -mab & 0 \end{bmatrix}$$

$$V^{+}(\varphi) = \begin{bmatrix} ma^{2} + Ml^{2} + ml^{2} - mlb\cos\varphi & mb^{2} - mlb\cos\varphi \\ -mlb\cos\varphi & mb^{2} \end{bmatrix}$$

After the heel strike the swing leg will become the new stance and the stance leg will the new swing leg that change can be computed using the following equation:

$$\theta^{+} = J\theta^{-} \quad \text{where } J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Therefore the matrix of transition function of the system can be written as:

$$y^{+} = T(\varphi)\, y^{-} \text{ where } T(\varphi) = \begin{pmatrix} J & 0 \\ 0 & K(\varphi) \end{pmatrix}. \tag{3}$$

The Complete Biped PDR Model

The complete biped passive dynamic robot can be described as follows:

$$\begin{cases} \text{Motion equation}: & \dot{y} = Ay \\ \text{Impact equation}: & y^{+} = T(\varphi)\, y^{-}, \qquad \text{when } \theta_{st} + \theta_{sw} = 2\alpha \end{cases}$$

Figure 3 shows the steps of walking cycle of PDR.

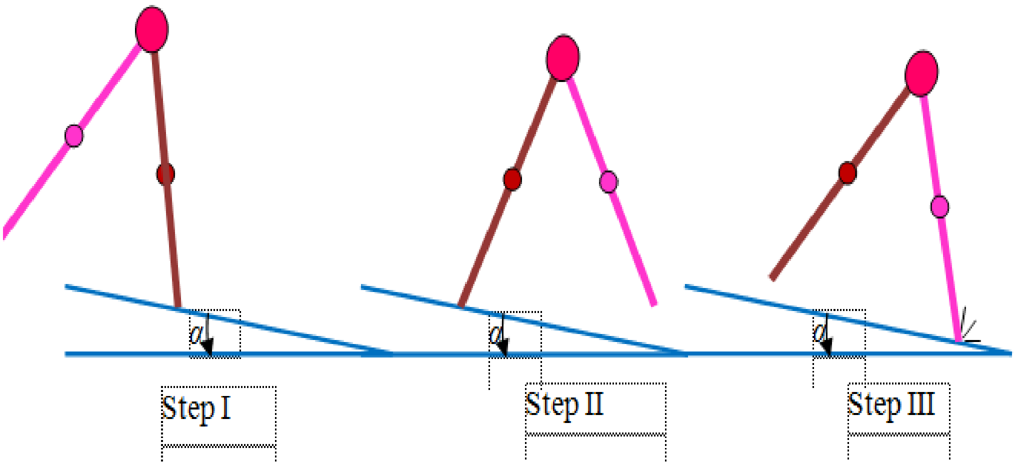

Figure 3: Walking Steps for PDBR

Analytic Solution

The system of PDBR is fusion of two distinct events: motion equation (2) and transition function (3). The analytical solution of the differential equation of motion (2) is given by the explicit equation

$$y(t) = e^{At} y_0$$

Where y_0 is the initial state-space position of PDBR. If τindicates the time of heel strike when the foot of swing leg contact the ramp-surface, then the state-vector y at the time of heel strike, can be expressed as

$$y(\tau) = e^{A\tau} y_0.$$

At the time of impact, the legs will alter their positions, to be precise, the swing leg will be the stance leg and the stance will turn out to be swing leg for the next step. This transformation will be done by the transition equation (3) and the state-space position of PDBR for the next step can be determined by

$$y^{+}(\tau) = T(\varphi)\, y(\tau).$$

Considering the state-vector $y^{+}(\tau)$ as the initial position state-vector for the subsequent step and repeating the above procedure with it.

Gait Stability

The theory of gait stability is difficult to define for a walking robotbut is critical for the performance analysis of the system. The usual definitions of stability of a system given by the Lyapunov around equilibrium pointare not immediatelyfit for the systems of walking robots.Understanding the complete dynamics of the robot over a symmetric gait cycle, it is more convenient to represent the dynamics in sense of phase space trajectories.In the phase space, a symmetric gait cycle is stable if it is beginningfrom a steady closed phase trajectory (orbit), then every finite perturbation leads to

another nearby trajectory of similar shape.As a result, we define the stability of a system in terms of its orbital stability as below (Goswami[6]).

Let us consider a second order system with state vector $y \in R^n$, initial conditions $y_{0.}$

$$\dot{y} = Ay$$

Considering the solutions of this equationas a trajectory in the vector space of the state vectors y, leads the following definitions:

Definition 1 (Orbit) For the above system, a positive limitset Ω of a bounded trajectory $y(t)\ \left(\left\|y(t)\right\| < \mu, \forall t > 0\right)$ is defined as:

$$\Omega = \left\{ p \in R^n, \forall \varepsilon > 0, \exists \{t_k\} / \left\| p - y(t_k) \right\| < \varepsilon, \forall k \in N \right\}$$

Ω is a closed orbit (limit cycle or non-static equilibrium)for the system. Note that for secondorder systems the only possible types of limit sets aresingular points and limit cycles. Let us consider thebehaviour of neighbouring trajectories in order to analyse the orbital stability.

Definition 2(Orbital stability) System trajectoryin the phase space R^n is a stable orbit Ω if ∀ε > 0, ∃δ > 0 such that

$$\left\| y_0 - \Omega(y_0) \right\| < \delta \Rightarrow \inf_{p \in \Omega} \left\| y(t) - p \right\| < \varepsilon, \forall t > t_0$$

That is, all trajectories are starting near the orbit Ω stay in itsneighbourhood.

Poincare Map

Poincare map is commonly used to study the passive walking and quite useful to analysestability of biped locomotion. The Poincare map reduces the the study of the stability of a periodic orbit to the study of the stability of a fixed point of the Poincare map.

The Poincare map is the mapping from the initial conditions $y_0 = \left(\theta_{st}, \theta_{sw}, \dot{\theta}_{st}, \dot{\theta}_{sw}\right)_0$ of one step to the next step of PDBR. The Poincare map $P : S \to S$ is defined as

$$y_{i+1} = P(y_i) = e^{A\tau} T(\varphi) y_i$$

where $S = \left\{ \begin{bmatrix} \theta_{st} & \theta_{sw} & \dot{\theta}_{st} & \dot{\theta}_{sw} \end{bmatrix} \middle| (\theta_{st} - \alpha) + (\theta_{sw} - \alpha) = 0 \text{ and } \theta_{st} - \theta_{sw} = \varphi \right\}$ is the Poincare section, the superscript "*i*" denotes step number and τ is the time of impact. A fixedpoint *y** of the Poincare mapis a point of Poincare section which satisfies (see figure 4)

$$P(y^*) = y^*.$$

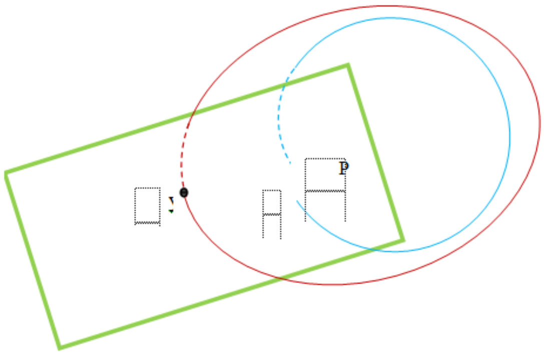

Figure 4 Fixed Point of Poincare Map

The stability of orbit in sense of fixed point of Poincare map is defined as follows:

Definition (Orbit Stability in Sense of Fixed Point) Orbit Ω is stable if for each ε>0, there exists a δ>0 such that

$$\|y_0 - y^*\| < \delta \Rightarrow \|P^n(y_0) - y^*\| < \varepsilon$$

But for the symmetric gait analysis, we are interested for$n$=1.

Simulation Results

For the simulation, we have assumed the values of parameters of PDBR which listed in the Table 2.

Table 2: List of Values Parameters

Symbols	Parameters	Values
a	The length of rod from the C.M. of Swing (Stance)leg to the foot	0.5 m
b	The length of rod from the C.M. of Swing (Stance)leg to the Hip	0.5 m
l	Length of rod=a+b	5 m
m	Mass of leg	1 kg
M	Mass of Hip	10 kg
α	Slope of ramp	0.035rad

With the above values of parameters, the fixed point of the Poincare map is y* = [-0.0360; 0.0360; -0.1870; -0.1896] and a time of heel-strike is τ=0.6857 sec. The orbit of a symmetric gait can be seen in figure 5

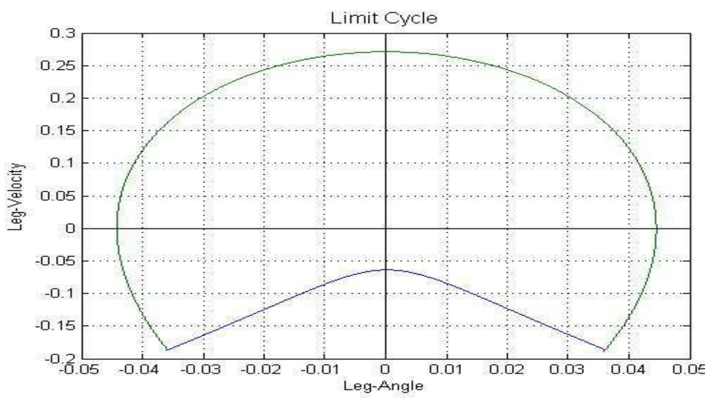

Figure 5: Phase Space Diagram of Symmetric Walk. This Diagram Corresponds to only One Leg of Biped Robot. It Shows the Swing Position, Stance Position and Impact Phase of the Leg of Biped Robot

The above figure 5 shows that all the three phases (swing phase, impact phase and stance phase) of a leg during the walking of PDBR with respect to the fixed point. To verify the stability of a fixed point of Poincare map, taking the small perturbations in the fixed point of Poincare map as follows.

Effect of Perturbation Infixed Point

Considering the small perturbation Δy in the fixed point$y$*, then there is a small deviation in P at the next step, which satisfiesthe linearized equation

$$y_{i+1} = P\left(y^* + \Delta y_i\right) = P\left(y^*\right) + \left(\frac{\partial P}{\partial y}\right)_{y=y^*} \Delta y_i$$

Using the following formula, we can find value of δ> 0 for the given value of positive ε for the stability criteria of orbit:

$$\Delta y = \left[I - J\right]^{-1}\left(P\left(y\right) - y^*\right).$$

The following figure 6 shows that simulation results of perturbation in the fixed point.

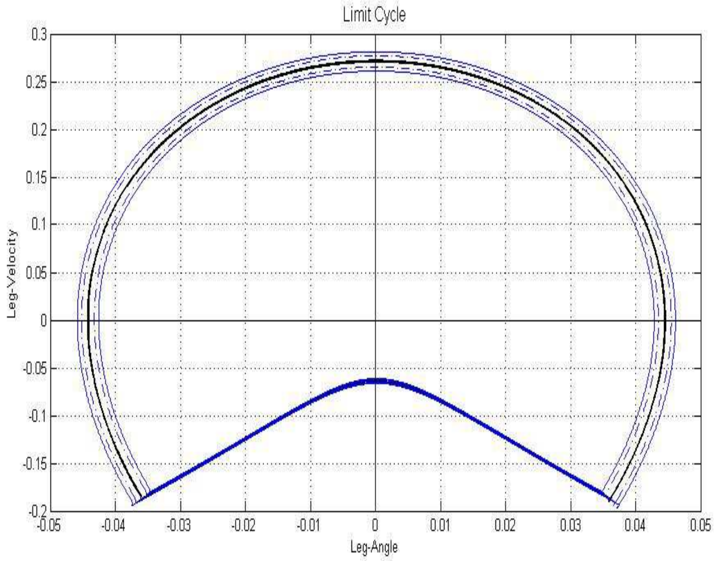

Figure 6: Phase Space Diagram of Perturbation in the Fixed Point of Poincare Map. This Diagram Shows that Perturbation Values Converge to Orbit of Fixed Point. Here the Area between Blue Lines Defines Stability

From figure 6, we examined that the small perturbation in the fixed point, will also generate the stable orbit in the vicinity of orbit of fixed point, that is, little disturbance still make stable walk for PDBR.

CONCLUSIONS

A passive dynamic robot without keen is an unactuated system. In this paper, we have established the theory of stability of symmetric gait of a passive dynamic robot over the inclined ramp.Wehave used the phase space diagrams and stability criteria of fixed point of the Poincare map in the stability analysis of the passive dynamic walking robot. The simulation results of analysis depicts that the robot is walking stable although initial state positions are perturbed. It shows the stepwise stability of the linearized model of the passive dynamic robot with respect to the initial condition, so it can be

considered as a local stability analysis of a symmetric gait. Our intension is to continue this research to analysis the global stability of the passive dynamic robot.

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